
Chapter 2

Introduction to Modern Measuring Techniques of Thermal Fluid Mechanics

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Course Contents

- Term project introduction & assignment (2/24)
- **Introduction to modern measuring techniques of thermal fluid mechanics (3/3, 3/10)**
- How to visualize your flow? (3/10, 3/17, 3/24, 3/31)
- **1st Mid-term discussions (4/21)**
- How to design and set up your experimental facility? (4/7, 4/14, 4/28, 5/05)
- How to quantitatively measure your system? (5/12, 5/19, 5/26, 6/02)
- **2nd Mid-term discussions (5/26)**
- Miscellaneous measurements (viscosity, surface tension, density, refractive index, thermal conductivity ...etc.) (6/09, 6/16)
- **Final Presentation & report (6/23)**

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Lecture Contents of Chapter 2

- What & Why thermal fluid mechanics ?
- Tools for solving problems
- The needs and design of modern measuring techniques of Thermal Fluid Mechanics
- Dimensional analysis & Π -theorem
- Modelling and Similarity
- Accuracy Analysis

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What do you think?

- (2021/02/25 中時新聞網) 理組「選? 台大生示警: 必修課看到「這2字」

<https://www.chinatimes.com/realtimenews/20210225006161-260405?chdtv>



現代熱流量測技術

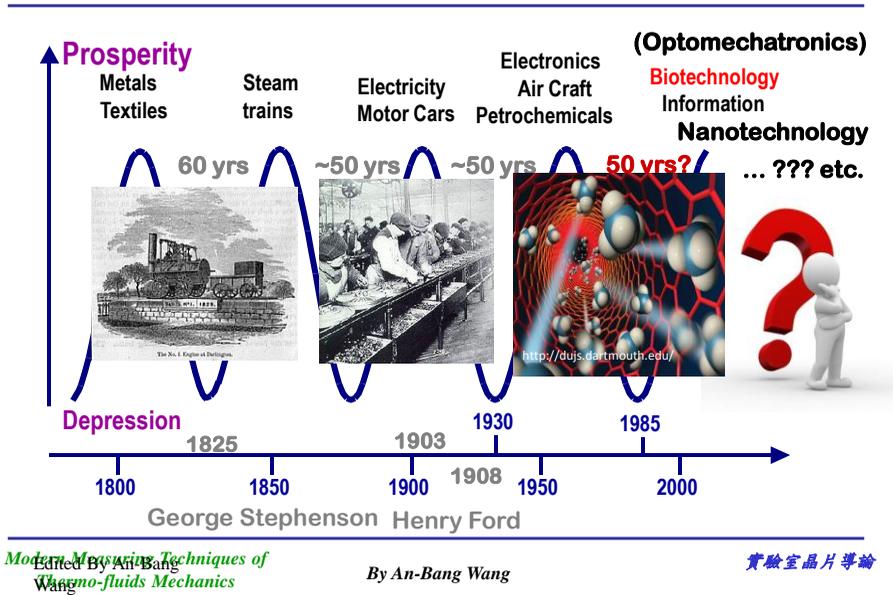
(Modern Measuring Techniques of Thermal Fluid Mechanics)

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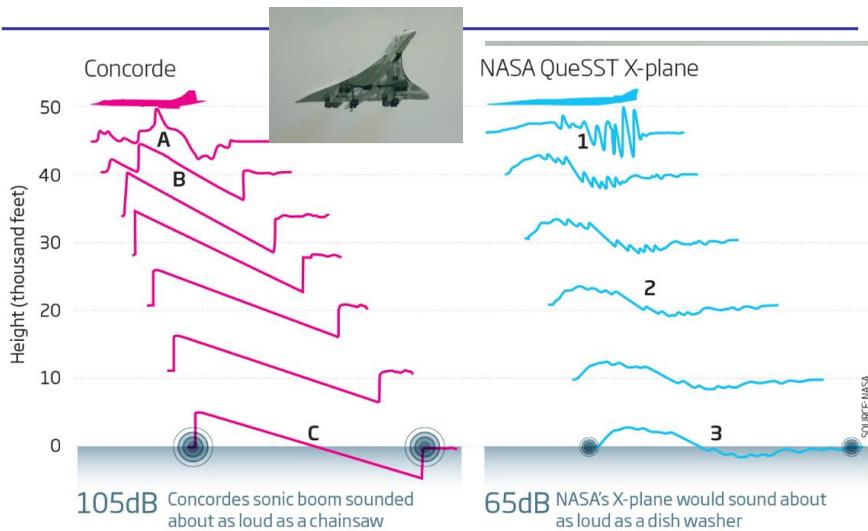
Trend of the world



What & Why Thermal Fluid Mechanics?



Flow Examples: Supersonic Airliner



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Thermal related Examples:

- Influence of $\langle \pm 2^\circ\text{C} \rangle$ ([2010年2月22日發布](#))
- **Human body temperature:** Normal: $37 \pm 0.5^\circ\text{C}$
(Ex: 中醫臨床的研究《算病》樓中亮中醫 [預防保健APP](#))
- Processing control in **chemical** engineering
- Processing control in **mechanical** engineering
- Processing control in **food** engineering
- Processing control in **agriculture**
- In many other process ...
- **Example of drop impact:**

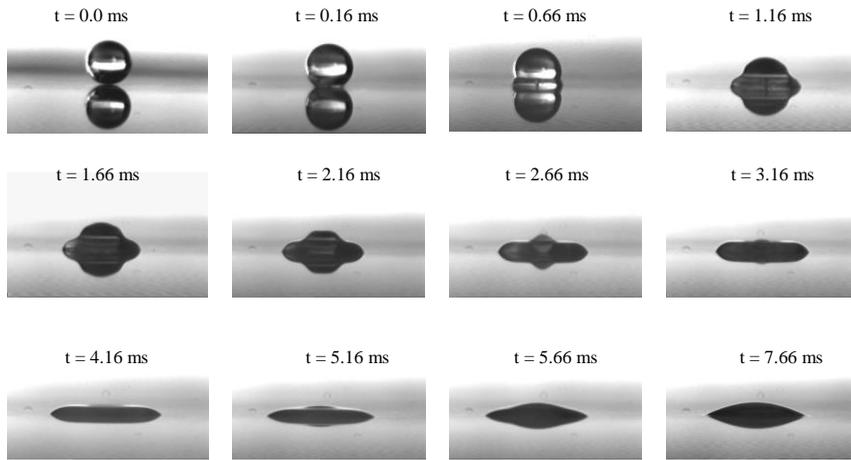
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Drop Impact Pattern (I) Completely Wet

(Ethanol, $D_p=550\mu\text{m}$; $Re=189$; $We=8.65$; $T_w=23^\circ\text{C}$)



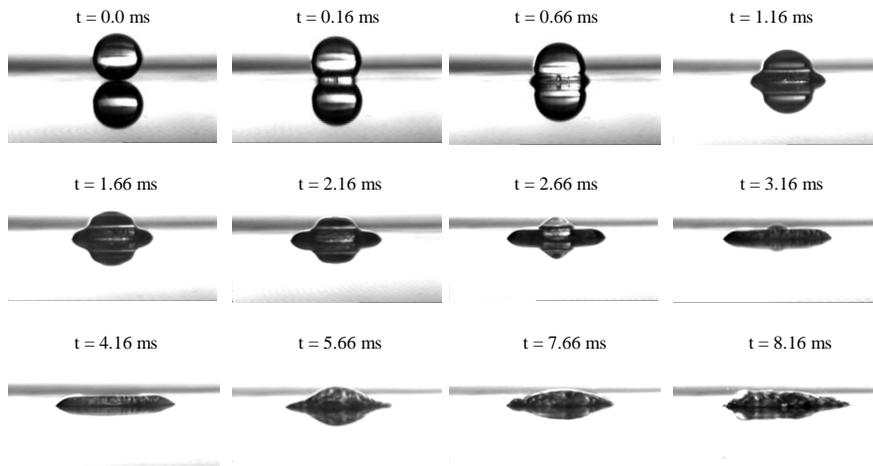
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Drop Impact Pattern (II) Wet Boiling

(Ethanol, $D_p=550\mu\text{m}$; $Re=189$; $We=8.65$; $T_w=81^\circ\text{C}$)



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A Close Look of Bubble Generation

Surface Temp. = $81 \pm 1.2 \text{ }^\circ\text{C}$
 $t = 6.16 \text{ ms}$

Surface Temp. = $62 \pm 1.0 \text{ }^\circ\text{C}$
 $t = 6.16 \text{ ms}$



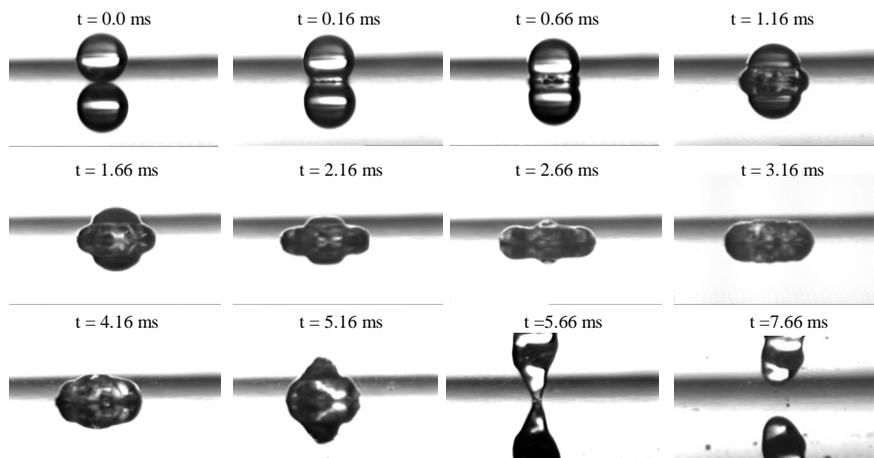
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Drop Impact Pattern (III) Transition

(Ethanol, $D_p=550\mu\text{m}$; $Re=189$; $We=8.65$; $T_w=116^\circ\text{C}$)



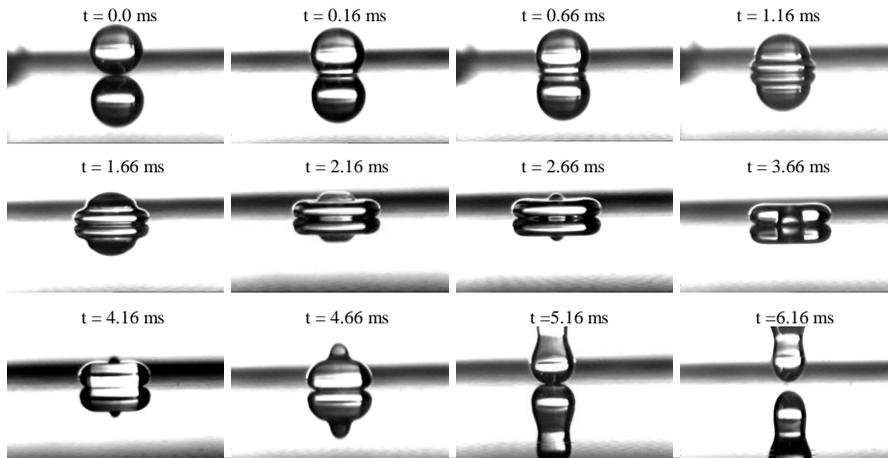
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Drop Impact Pattern (IV) Dry Rebounding

(Ethanol, $D_p=550\mu\text{m}$; $Re=189$; $We=8.65$; $T_w=130^\circ\text{C}$)



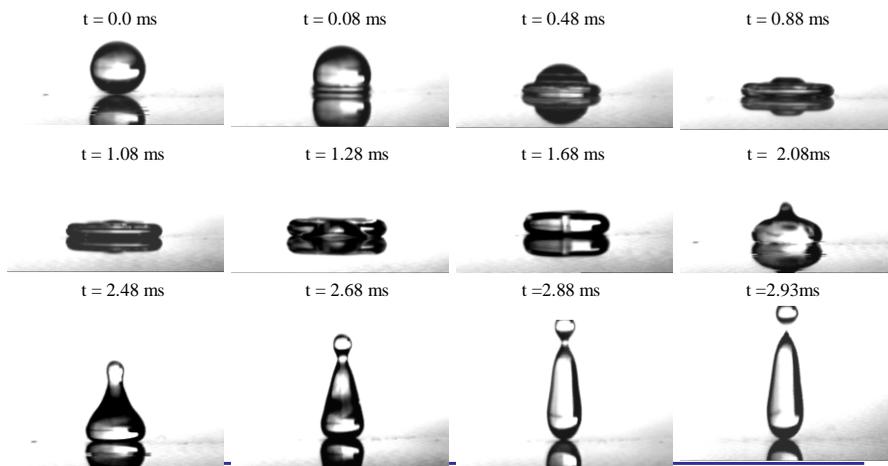
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Drop Impact Pattern (V) Satellite Dry Rebounding

(Ethanol, $D_p=550\mu\text{m}$; $Re=881$; $We=188.33$; $T_w=138^\circ\text{C}$)



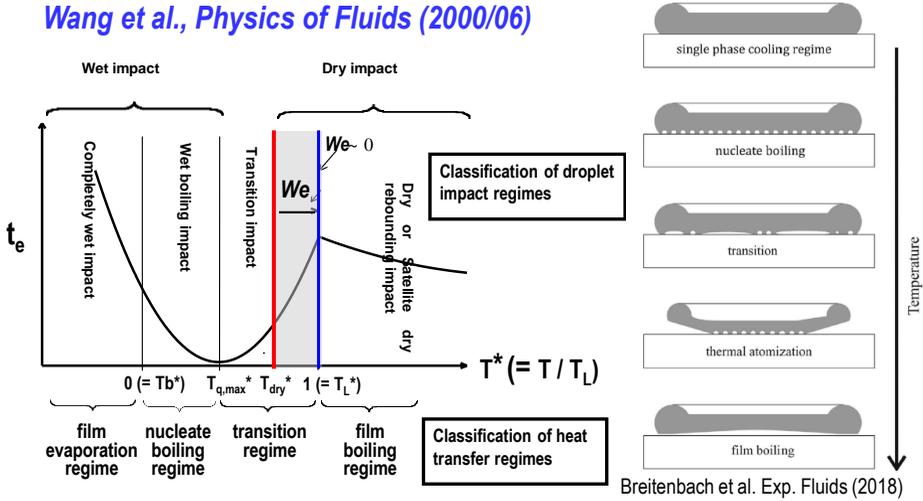
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Classification of drop impact regimes

Wang et al., *Physics of Fluids* (2000/06)



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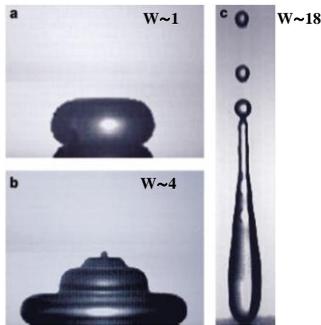
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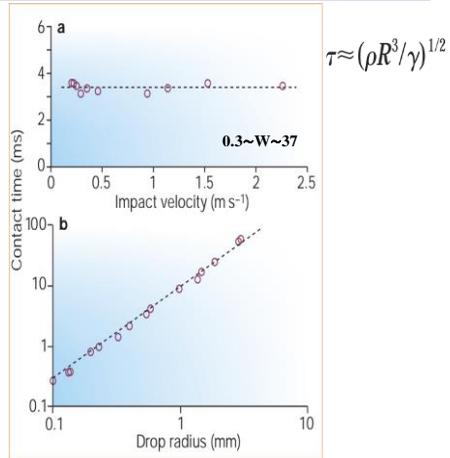
Contact time of a bouncing drop

Denis Richard*, Christophe Clanet†, David Quéré *NATURE* | VOL 417 | 2002

$$W = \rho V^2 R / \gamma$$



The contact time is independent of the details of the impact.



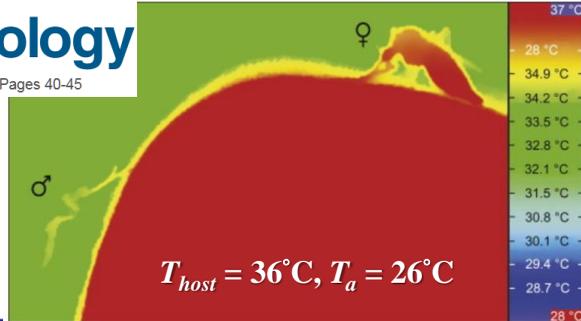
The contact time does not depend on the impact over a wide range of velocities. (20 – 230 cm/s)

Temperature Control of animals

- 吸血維生的蚊蟲都是大胃王。蚊子在飽足一頓血之後，體型可以變成進食前的兩到三倍大；壁蝨、錐鼻蟲，進食後更是可以脹大十倍至一百倍。(https://case.ntu.edu.tw/blog/?p=30156)
- 節肢動物是變溫動物(環境溫度等於體溫)。蚊子吸完血之後，體溫可以在一分鐘內急速上升攝氏 10 到 15 度。

Current Biology

Volume 22, Issue 1, 10 January 2012, Pages 40-45

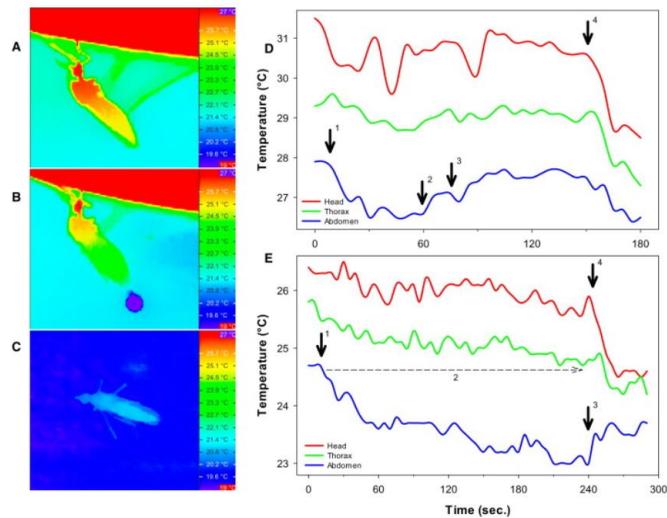


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Temperature Control of animals



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What did you learn?

- What are the difference between them (experiment in lab course and term-projects? High I.F.-journal and others?)
- Is the conducting process the same? Or different?
- Which one is simpler?
- What do you want to be?

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IA-products



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Thermal Solutions & Measurement



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影像顯示科技(FPD Technology)

- 視覺為五覺之首，是接受資訊與知識之大門



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Bio-technology



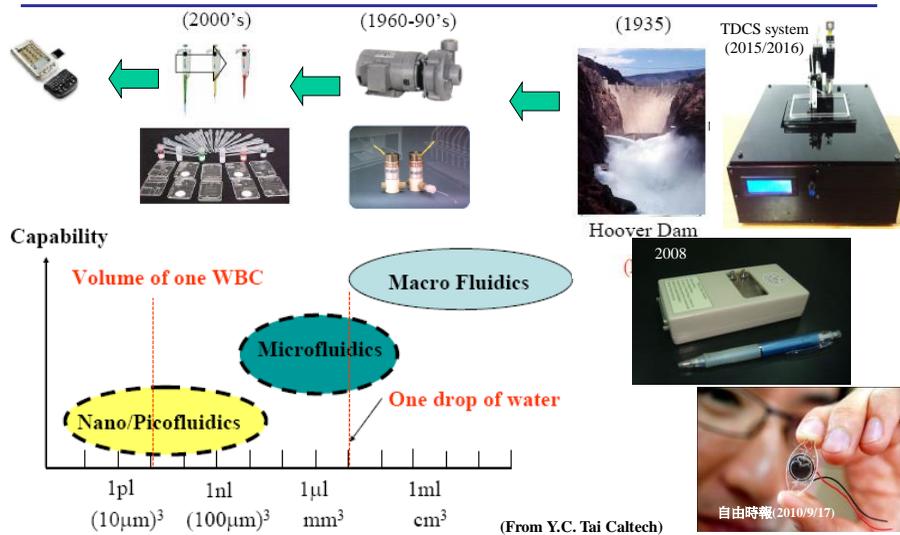
Wang et al. 2011 Electrophoresis

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Microfluidics and Microfluidic Platform

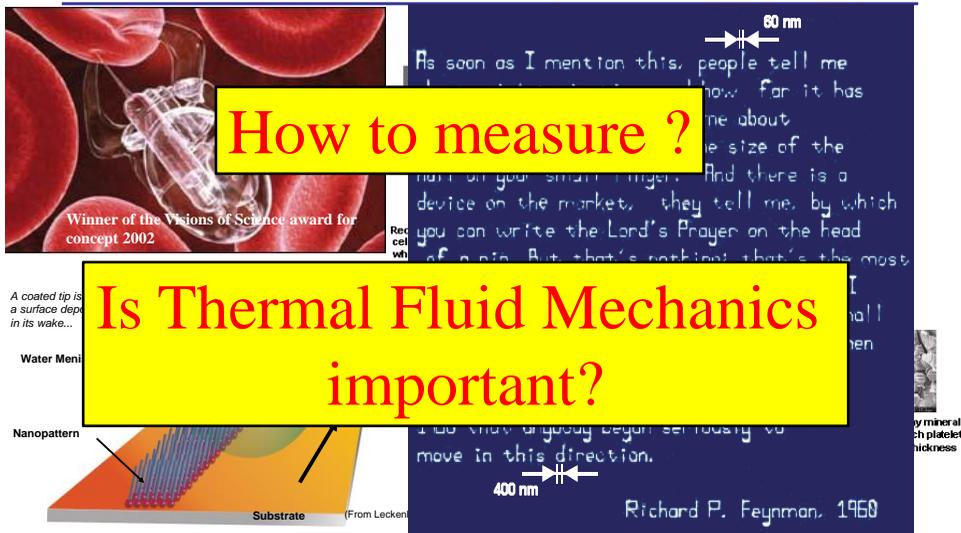


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Nano-technology



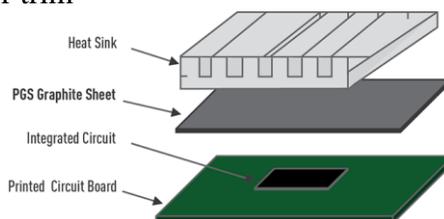
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Resource/Outsourcing from the market

- PGS: Pyrolytic Graphite Sheet
- Thermal conductivity: 700 to 1950 W/(m-K) (2- 5 times of copper [400W/(m-K)] , 3 - 8 times of aluminum [237W/(m-K)])
- High stability (withstand up to 400 °C), no deterioration with age
- Simultaneous solution for thermal and electromagnetic wave problems
- Thin, flexible and easy to cut or trim
- Withstands repeated bending
- Low thermal resistance
- RoHS (Restriction of Hazardous Substances) and REACH compliant



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Courses of Thermal Fluid Mechanics/Science

- **Thermo-fluids, Thermal/Fluids, Thermal Fluid, ...**
- **Thermal courses:**
 - Thermal-Fluids Engineering I, II (MIT)
 - Thermodynamics

How about Fluid Mechanics courses?

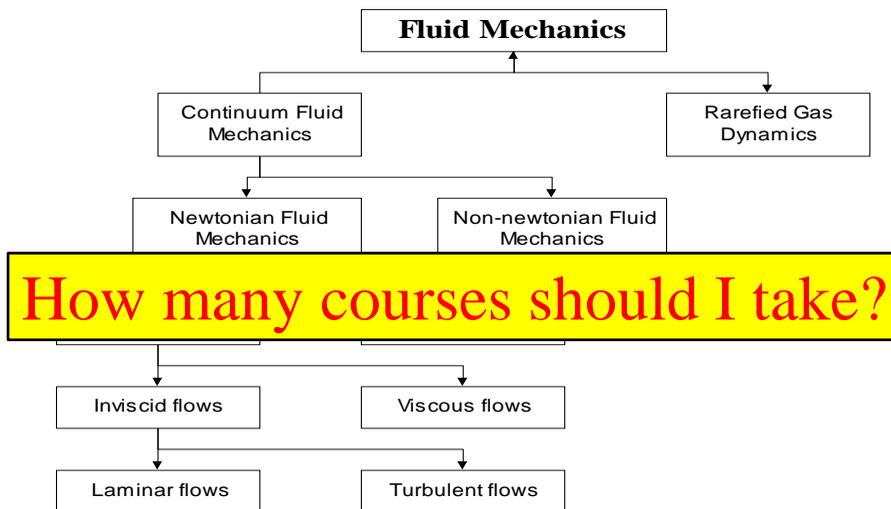
- Transport Phenomena
- Engineering Analysis
- Mathematical Methods in Applied Mechanics
- Advanced Thermodynamics

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Scope of Fluid Mechanics

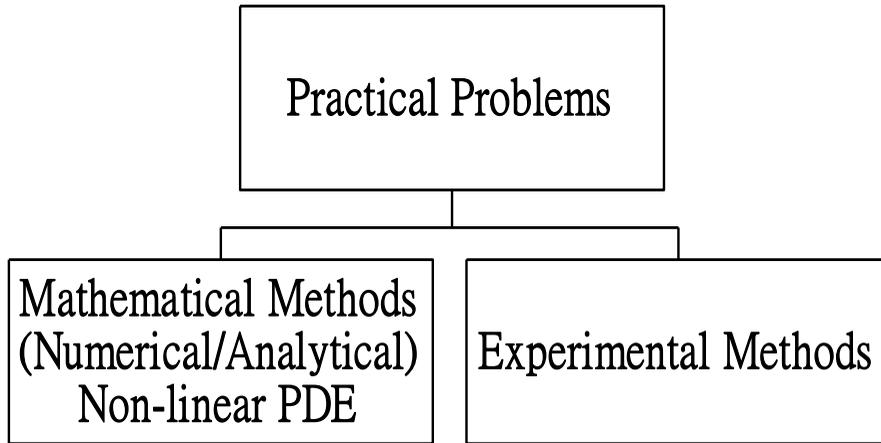


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Tools for Solving Problems

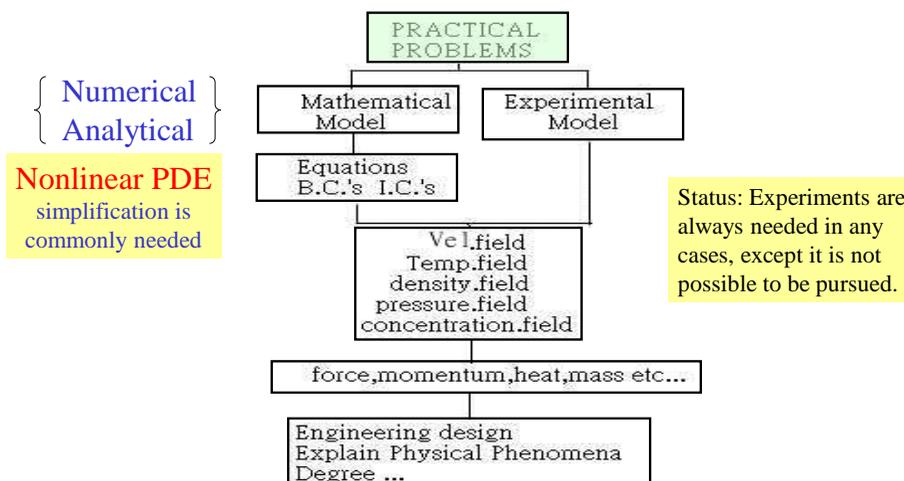


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Approaches to solve problems of Fluid Mechanics



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Experiments & Simulations



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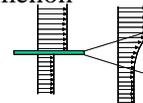
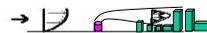
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Why experiments ?

The need for experiments in fluid mechanics arises from a variety of aims :

- **Basic research**
 - Extend physical understanding of a particular flow phenomenon
 - Test new theoretical results
 - Verify numerical models (e.g. in CFD)
- **Model studies** (inexpensive compared to prototype tests in most cases)
 - Investigate an unknown flow situation or test new apparatus designs
 - conduct a systematic parameter study and/or optimization
 - establish scaling laws
- **Flow Measurement**
 - E.G. Volume flow rate, drag, lift ...etc
 - Measurement of a quantity for control feedback purposes.



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Design of an Experiment (I)

In designing an experiment, a number of questions must be asked :

- **Which quantities** are important to measure (both independent and dependent)?
 - This is answered partly through the *aim of the experiment* and partly through *experience*.
 - *Dimensional analysis* is *indispensable* for complex systems.
- In **what range** will the measured quantities vary ?
 - This information, and information about the required accuracy is necessary for the choice of measuring technique.
- **Which quantities must be controlled ?**
 - operating conditions must be well defined in order to control the experiments to be *repeatable*, *stationary* and simulated to the given conditions.

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Design of an Experiment (II)

- What is the required **dynamic response** of the measuring instrument?
 - Is any correction or compensation necessary ?
- What is the **measurement time** ?
 - For time mean averaged quantities, a given statistical variance should be specified.
- Has this experiment already been performed ?
- Does a theoretical solution exist ?

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Check points I : How about my case?

- *Have I a clear topic to start in this?*
- Has this experiment already been performed ?
- Does a **theoretical solution** exist ?
- What is my *novelty*?

Dimensional analysis

Dimensional analysis facilitates the interpretation and extends the range of application of experimental data by correlating them in terms of dimensionless groups. Its most serious limitation is that it gives **no** information about the nature of a phenomenon. Rather it is necessary to know before hand which variables influence the phenomenon.

Typically ,the primary dimensions are chosen as

- M : mass
- L :length
- T :time
- θ : temperature

Why dimensional analysis (I)?

Dimensional analysis (**DA**) is enormous time- and money-saving !

Example: force on an immersed body

$$F = f(L, V, \rho, \mu)$$

generally speaking, we need 10 experimental points to define a curve, this means that we need

$$10 \times 10 \times 10 \times 10 = 10^4 \text{ experiments.}$$

assuming NT \$10 /experiment we need 10^5 NT - dollars

and 0.1 day /experiment, we need 10^3 days = 2.7 years !!!

However, by using **DA**

$$C_f = F / \rho V^2 L^2 = g(\rho V L / \mu) = g(Re)$$

Nothing is lost, but with

NT \$ 10 /experiment x 10 experiments = 100 NT-dollars

and 0.1 day /experiment x 10 exp. = 1 day!!

It's a big difference !

Why dimensional analysis (II)?

2. **DA** helps thinking and planning (not only experiment but also theory)!

It suggest variable which can be discarded and often give a great deal of insight into the form of the physical relationship.

3. **DA** provides *scaling law* (or similarity) which may convert data from a cheap, small model into design information for an expensive, large prototype.

- A method for describing dimensionless parameter is generally credited to E. Buckingham in 1914, and is commonly called "**Buckingham Π -theorem**".
- From the governing Equations, boundary conditions, we may also get the governing dimensionless parameters

Dimensionless Groups in Fluid Mechanics (I)

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$Re = \frac{\rho UL}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Always
Mach number	$Ma = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$Fr = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$We = \frac{\rho U^2 L}{\gamma}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Cavitation number (Euler number)	$Ca = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$Pr = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$Ec = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$\gamma = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow

White (1986)

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Dimensionless Groups in Fluid Mechanics (II)

Parameter	Definition	Qualitative ratio of effects	Importance
Strouhal number	$St = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$Gr = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2} \rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2} \rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics

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Buckingham π -theorem

Process :

1. list down the all variables involved in the problem

($n = ?$) $F(B_1, B_2, B_3, \dots, B_n) = C$ (Most critical and difficult process!)

2. choose the primary j variables, which **cover the whole needed dimensions**, but do not form a π -product

($j = ?$) $B_1, B_2, B_3, \dots, B_j$ (In fluid mechanics M, L, T, Θ !)

3. Add each additional variable to your j variables to form a power product, then find the exponents which make the $n-j$ product dimensionless ,

$$\Pi_1 = B_1^{a_{1,1}} B_2^{a_{1,2}} \dots B_j^{a_{1,j}} B_{j+1}$$

$$\Pi_2 = B_1^{a_{2,1}} B_2^{a_{2,2}} \dots B_j^{a_{2,j}} B_{j+2}$$

$$\Pi_{n-j} = B_1^{a_{n-j,1}} B_2^{a_{n-j,2}} B_j^{a_{n-j,j}} B_n$$

4. we get finally $F_1(\Pi_1, \Pi_2, \dots, \Pi_{n-j}) = C_1$

(the reduction of variables : j)

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Example of Buckingham π -theorem (I)

Consider the problem of pressure drop in steady pipe flow

(1) $F = F(\Delta p, V, L, D, \rho, \mu, e) = C \Leftrightarrow n = 7$

(2) choose primary j variables: $V, D, \rho \Leftrightarrow j = 3$

(Which one is better? It's a matter of taste, custom and user's choice.)

(3) Find the $n-j$ Π : $\Leftrightarrow n-j = 7-3 = 4$

$$\Pi_1 = V^{a_{1,1}} D^{a_{1,2}} \rho^{a_{1,3}} \Delta p$$

$$\Pi_2 = V^{a_{2,1}} D^{a_{2,2}} \rho^{a_{2,3}} L$$

$$\Pi_3 = V^{a_{3,1}} D^{a_{3,2}} \rho^{a_{3,3}} \mu$$

$$\Pi_4 = V^{a_{4,1}} D^{a_{4,2}} \rho^{a_{4,3}} e$$

$$\Pi_1 = V^{a_{1,1}} D^{a_{1,2}} \rho^{a_{1,3}} \Delta p$$

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Example of Buckingham π -theorem (II)

$$\begin{aligned}\Pi_1 &= M^0 L^0 T^0 = V^{a_{1,1}} D^{a_{1,2}} \rho^{a_{1,3}} \Delta p \\ &= (LT^{-1})^{a_{1,1}} (L)^{a_{1,2}} (ML^{-3})^{a_{1,3}} (ML^{-1} T^{-2})\end{aligned}$$

for M: $0 = a_{1,3} + 1$

L: $0 = a_{1,1} + a_{1,2} - 3 \times a_{1,3} - 1$

T: $0 = -a_{1,1} - 2$

so that $a_{1,1} = -2, a_{1,2} = 0, a_{1,3} = -1$

$$\Pi_1 = M^0 L^0 T^0 = V^{-2} D^0 \rho^{-1} \Delta p = \Delta p / \rho V^2$$

Similarly, $\Pi_2 = L/D, \Pi_3 = \mu/\rho V D (= Re), \Pi_4 = e/D$

$$\Rightarrow F_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = F_1(\Delta p / \rho V^2, L/D, \mu/\rho V D, e/D) = C_1$$

or $\Delta p / \rho V^2 = F_2(L/D, \mu/\rho V D, e/D)$

However, DA says nothing about the functional form of F1 or F2!!

Modelling and Similarity

$$\begin{aligned}\Pi_1 &= F(\Pi_2, \Pi_3, \dots, \Pi_k) \\ \text{model} \quad \Pi_{2m} &= \Pi_{2p} \quad \text{prototype} \\ \Pi_{3m} &= \Pi_{3p} \\ &\dots \\ \Pi_{km} &= \Pi_{kp} \\ \Rightarrow \Pi_{1m} &= \Pi_{1p} \quad \Rightarrow \text{Complete Similarity}\end{aligned}$$

But in engineering, instead of complete similarity, We consider:

Geometric similarity (L-scale)

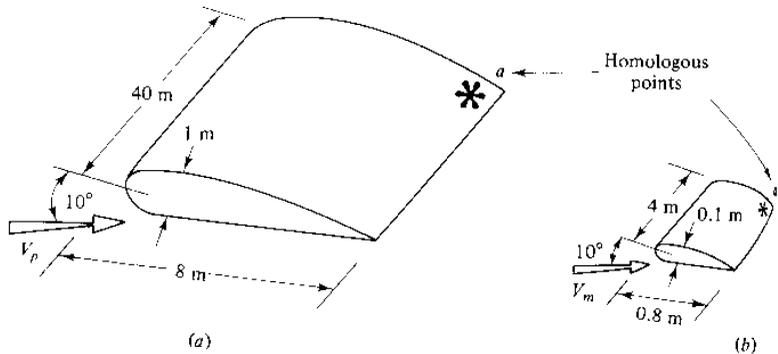
Kinematic similarity (L- & t-scale)

Dynamic similarity (L- & t- & m-scale)

(Thermal similarity)

Geometric Similarity

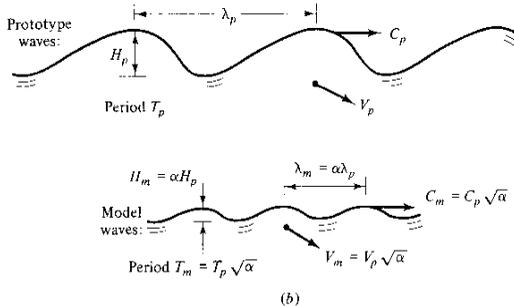
Geometric Similarity:
It requires all body dimensions in all three coordinates have the same linear scale ratio



Kinematic Similarity

Kinematic Similarity
It requires the model and prototype have the same length-scale ratio (geometric similarity) and also the same time scale ratio

Inviscid free-surface flow



$$\lambda_m/\lambda_p = \alpha = H_m/H_p$$

since $Fr_m = Fr_p$

$$V_m^2/g\lambda_m = V_p^2/g\lambda_p$$

$$\Rightarrow V_m/V_p = (\lambda_m/\lambda_p)^{1/2} = \alpha^{1/2} = \beta$$

α & β are dependent!

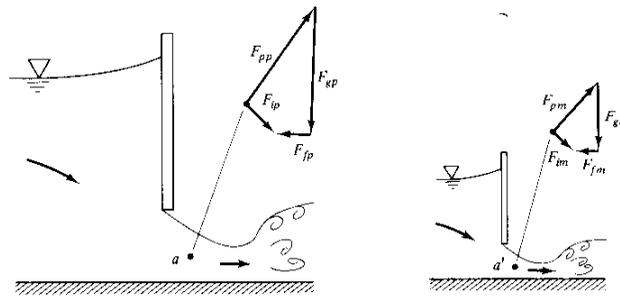
also $c_m/c_p = \alpha^{1/2}$

period: $T_m/T_p = \alpha^{1/2}$

Dynamic Similarity

- Dynamic Similarity

It requires the same *length-scale ratio*, *time scale ratio* and also *force-scale ratio*



$$F_{p(\text{ressure})} + F_{g(\text{ravity})} + F_{f(\text{riction})} = F_{i(\text{neria})}$$

$$Fr_p = Fr_m \quad Re_p = Re_m$$

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Discrepancies in model-prototype similarity (I)

- The perfect dynamic similarity is more of a dream than a reality because true equivalence of characteristic dimensionless parameters can be achieved only by dramatic changes in fluid properties, whereas in most model testing is simply done with air and water, the cheapest fluids available.
- Example: consider hydraulic model testing with free surface:

Geometric similarity:

$$\lambda_m / \lambda_p = \alpha = 1/10$$

Froude number (Fr):

$$V_m^2 / g \lambda_m = V_p^2 / g \lambda_p \Rightarrow V_m / V_p = (\lambda_m / \lambda_p)^{1/2} = \alpha^{1/2} = 0.32$$

Reynolds number (Re):

$$V_m \lambda_m / \nu_m = V_p \lambda_p / \nu_p \Rightarrow \nu_m / \nu_p = (\lambda_m / \lambda_p)^{3/2} = \alpha^{3/2} = 0.032$$

$$\nu_p = 1 \text{ mm}^2/\text{s} \text{ for water } (\nu_{\text{mercury}} = 0.12 \text{ mm}^2/\text{s})$$

\Rightarrow Re-similarity is unavoidably violated!!

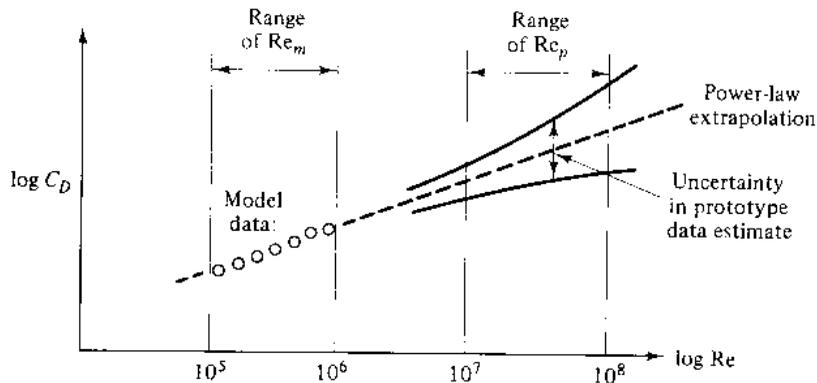
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Discrepancies in model-prototype similarity (II)

In practice, water is used for both model and prototype!



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Check points II: What is my case?

For my experiment

- Which quantities are important to measure (both independent and dependent)?
- Which quantities must be controlled ?
- In *what range* will the measured quantities vary? (Dynamic/Kinematic/Geometric Similarity?)
- What is the required **dynamic response** of the measuring instrument?
- How to determine the **measurement time**?

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Terminology for Error Analysis

Terminology

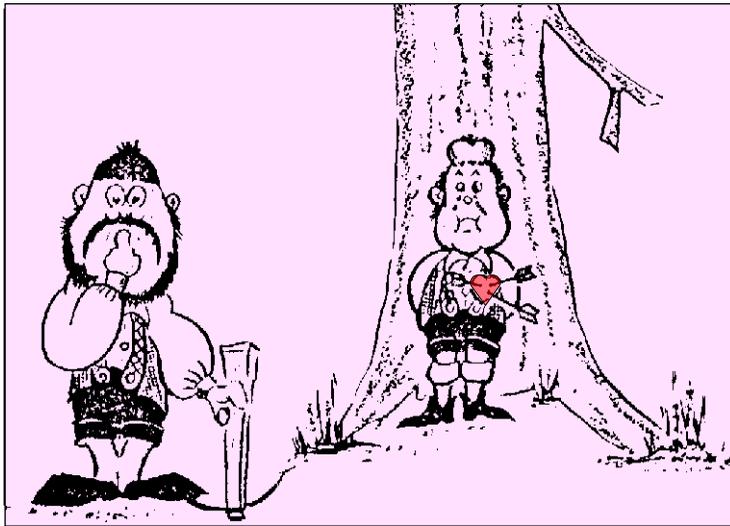
- **Error**: deviation of the reading from a known input
- **Accuracy**: Error, usually expressed as a percentage of full-scale reading, for industrial or laboratory instruments.
- **Uncertainty**: range within the error is likely to fall with specified confidence limits (or fiducial limit).
- **Precision** (Repeatability):
 - **reproducibility** of the reading for a given input.
 - an instrument can be precise, but **not** calibrated or misused
 - accuracy of an instrument cannot be better than its precision.
- **Traceability**: The ability to trace the accuracy of a standard back to its ultimate source in the fundamental standards (e.g., NIST)
- **Sensitivity** : ratio of instrument output to input.

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Precision(repeatability) & Accuracy



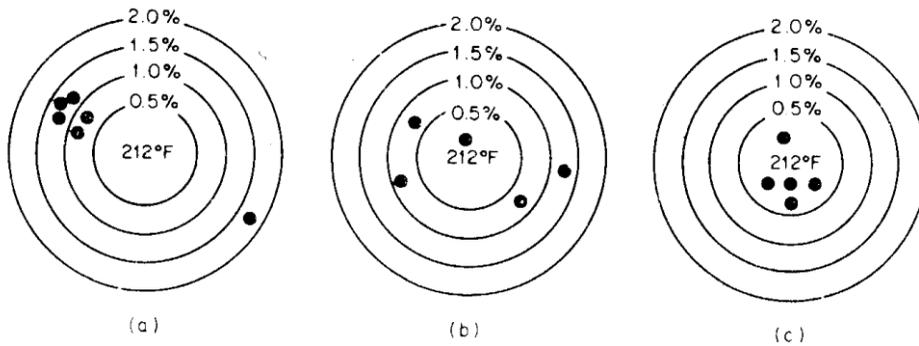
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Precision, Bias error & Accuracy (I)

Calibration Data for Three Temperature-Measuring Devices



†Possible illegitimate error or an outlier.

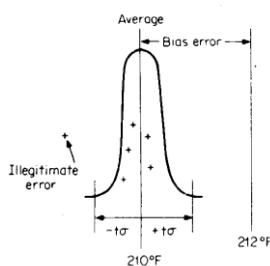
(From Miller, 1983)

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Precision, Bias error & Accuracy (II)



(From Miller, 1983)

With the outlier omitted, the average of the reading

$$\bar{I} = \frac{\sum I_i}{n} = \frac{(210.1+210.0+209.8+210.2+209.9)}{5} = 210.0$$

the standard deviation is:

$$\sigma = \left[\frac{\sum (I_i - \bar{I})^2}{n-1} \times 100\% \right]^{0.5}$$

$$= \left[\frac{(0.0023+0+0.0090+0.0090+0.0023)}{(5-1)} \right]^{0.5} = 0.0753\%$$

∴ the **precision** at the 95% confidence level is then:
 $\sigma_p = t_{st} \sigma$, where t_{st} is two-tailed student's t-value, could be found from table ($t_{st} = 2.776$ for $n=5$).

∴ the precision is then $\sigma_p = 2.776 \times 0.0753\% = 0.21\%$

The **direction bias error** is

$$B = \frac{\bar{I} - I_t}{I_t} \times 100\% = \frac{(210 - 212)}{212} \times 100\% = -0.94\%$$

∴ the **accuracy** is $A_{cc} = B \pm \sqrt{\left(1 + \frac{1}{n}\right) \sigma_p} = -0.7\% \sim -1.2\%$

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Error Analysis (I)

- Almost all fluid flow measurements are indirect. (e.g. Pitot-tube for pressure, hot-wire for velocity ...)
- In experimental work, errors of two different types can occur :
 - Systematic Errors
 - poorly adjusted instruments
 - improper calibration
 - false instrument specification
 - incorrect or biased statistical estimators
 - Statistical or random errors
 - improper reading from a scale
 - statistical variance of the measured quantity.
- Some errors can be corrected or controlled, others (uncertainty) cannot. All must be estimated !

Error Analysis (II)

- The influence of the errors on the end result can be determined using the error propagation rule from Gauss , i.e. ,
A quantity $\Phi = f (P_1, P_2, \dots)$ dependent on individual parameters P_i , and with individual uncertainties δp_i , the uncertainty $\delta\Phi$ could be written as

$$\delta\phi = \left\{ \left(\frac{\partial\phi}{\partial p_1} \delta p_1 \right)^2 + \left(\frac{\partial\phi}{\partial p_2} \delta p_2 \right)^2 + \dots \right\}^{1/2}$$

In applying this relation, Gaussian error distributions are assumed.

Error analysis (III)

The Prandtl tube is used to measure flow velocity:

$$\Delta P = P_1 - P_2$$

Where

R: gas constant,

T_1 : gas temperature,

P_1 : stagnation pressure

g: acceleration due to gravity

$$U = \sqrt{\frac{2\Delta P R T_1 g}{P_1}}$$

The uncertainty in U becomes

$$\delta u = \left\{ \left(\frac{1}{4} \frac{2RT_1g}{(\Delta P)p_1} \right) \delta \Delta P^2 + \left(\frac{1}{4} \frac{2\Delta P R T_1 g}{p_1^3} \right) \delta p_1^2 + \left(\frac{1}{4} \frac{2\Delta P R g}{p_1 T_1} \right) \delta T_1^2 \right\}^{1/2}$$

The normalized uncertainty is then

$$\frac{\delta u}{u} = \left\{ \left(\frac{1}{2} \frac{\delta(\Delta P)}{\Delta P} \right)^2 + \left(\frac{1}{2} \frac{\delta(p_1)}{p_1} \right)^2 + \left(\frac{1}{2} \frac{\delta(T_1)}{T_1} \right)^2 \right\}^{1/2}$$

Clearly, a 0.1% accuracy in reading the temperature is not worthwhile, if for instant the ΔP can only be determined with 5% accuracy !

Some notes for electronics

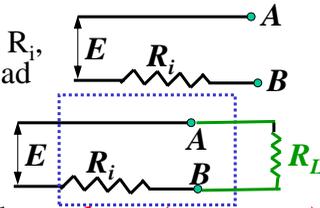
- Impedance Matching

It is continually necessary to connect various electronic instruments in different combinations

Every instrument has an internal resistance R_i , in series with an externally connected load resistance R_L ,

The voltage

$$E_{AB} = E \frac{R_L}{R_L + R_i}$$



- If E_{AB} is to be measured, R_L should be chosen **large (e.g. scopes)**.
- If power is to be transmitted

$$P = E_{AB}^2 / R_L \text{ or } P = E^2 R_L / (R_L + R_i)^2$$

For a maximum transmission : $dP/dR_L = 0$ then $R_L = R_i$

i.e. resistive part of impedance should be match. The inductive and capacitive terms must also be considered when dynamic response is important.

Wheatstone bridge

The Wheatstone bridge is a basic building block of many measuring instruments. It allows precise measurement of minute change in resistance, capacitance or inductance. It has many applications in

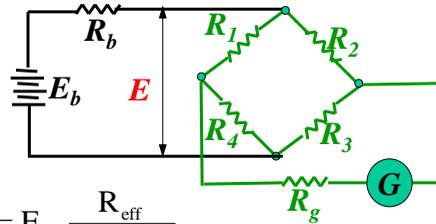
- hot-wire anemometry
- strain gauges
- Inductive pressure transducers or condenser microphones

- effective bridge resistance:

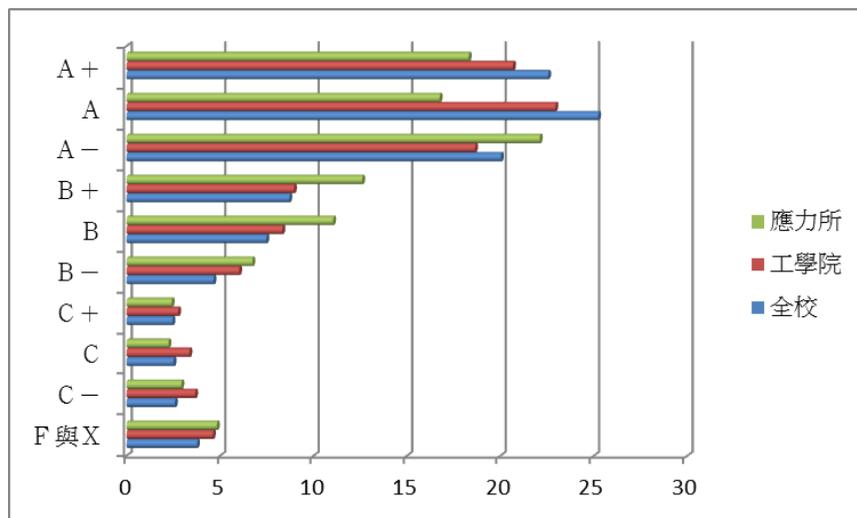
$$R_{\text{eff}} = \frac{(R_1+R_4)(R_2+R_3)}{R_1+R_2+R_3+R_4}, \quad E = E_b \frac{R_{\text{eff}}}{R_{\text{eff}}+R_b}$$

- Voltage measured at galvanometer:

$$E_g = E \left(\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3} \right)$$



Statistic score data of NTU



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Topics of Term-Project

- 蔡俊雄: Mask optical optimization for metling-laser application
- 鄭聿: On the drop coalesce mechanism
- 邱鴻年: Design & development of a novel micro-reactor system
- 鄭珮好: Development of medical contact lens & its measurement
- 趙士懿 : Development of medical contact lens & its measurement